An Analysis of Fees in Funds of Funds

15.451 ProSeminar in Financial Engineering
Rosenfeld Project

Ola Ayaso
Cabot Henderson
Andrew Henwood
Evan Schwartz
Ilya Zusman

November 2, 2006
Contents

1 Introduction 1
  1.1 Project Description and Goals 1
  1.2 Hedge Funds and Funds of Funds 1
  1.3 Fees 2
  1.4 Incentive Fees as Options 3

2 Model 4
  2.1 Returns, Fees and Sharpe Ratio 6
  2.2 Valuation of Incentive Fees as Options 8
  2.3 Deadweight Cost 9

3 Simulation 10
  3.1 Model Parameters 11

4 Results 13
  4.1 Correlation 14
  4.2 Correlation and Volatility 15
  4.3 Correlation and Number of Funds 16

5 Discussion and Conclusions 18
1 Introduction

1.1 Project Description and Goals

Our project explores the effects of management and incentive fees charged funds of funds (FOFs) on the returns of investors. Specifically, we analyze the typical fee structure through use of Monte Carlo simulations. We also investigate the effects of the correlation of individual hedge fund (HF) returns, the volatility of expected returns and the number of HFs in the FOF portfolio on the total cost of the fee structure to the investor.

1.2 Hedge Funds and Funds of Funds

The HF asset class has grown explosively in size and importance over the past decade. Comprising over 10,000 funds with roughly $1.4 trillion under management [1], the HF sector has attracted increased attention from investors drawn to the gaudy returns and diversification benefits. A HF is a private investment partnership, capitalized by qualified investors (traditionally wealthy individuals and institutions) and run by managers who can operate outside the constraints of traditional money managers.

Not having to register with the SEC, HFs are lightly regulated. Most traditional money managers have very tight restrictions on the assets that they can invest in and are forbidden from certain investing activities such as short-selling or using leverage. HFs typically focus on one particular market sector, with the Credit Suisse/Tremont Hedge Fund Index identifying fourteen major strategies to categorize them: Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Equity Market Neutral, Event Driven, Distressed, Multi-Strategy, Risk Arbitrage, Fixed Income Arbitrage, Global Macro, Long/Short Equity, Managed Futures and Multi-Strategy.

Given the low level of transparency, the complexity of the trades undertaken, and the frequent use of leverage, HFs have acquired the reputation of being a relatively risky asset class, although this depends largely on the goals and methods of the individual HFs. High profile examples of extreme HF underperformance have heightened this characterization, either from fraudulent practices or trades that have gone against the managers. In 2005, Bayou Management, LLC imploded after it was revealed that the $450 million fund had been falsifying its returns for years. Just months ago, Amaranth announced losses of $6 billion after investments in natural gas contracts went against the Greenwich-based fund.

In part to combat these fears and in part to extend the diversification benefits of investing broadly
across this asset class, investors have increasingly turned to FOFs. Although market data is inconsistent and hard to find, it is estimated that FOFs control about a third of HF assets. FOFs have clearly grown substantially in influence at the beginning of 2000, FOFs comprised only 15 percent of HF assets. These investment vehicles comprise of portfolios of individual HF investments with their own management and incentive fees. Through FOFs investors are spreading their money across a wide range of individual HFs, usually with different styles and strategies. FOFs claim to perform due diligence to find the best HF managers, provide control over management risk, and facilitate diversification by investing in numerous HF styles. A typical FOF invests in anywhere from 35-50 underlying HFs. Since most individual funds require large minimum investments, investors also find FOFs the only practicable way to invest in the asset class without exposing themselves to extreme idiosyncratic risk.

1.3 Fees

Since HF managers claim to have the ability to generate consistent alpha, it stands to reason that investors are expected to pay dramatically for managers’ expertise. Customarily, investors pay a management fee derived from a percentage of the assets under management and an incentive fee that comes from a proportion of the trading profits. The current industry standard is a management fee of 2 percent of assets under management and an incentive fee of 20 percent of profits (or “2 and 20”). Even this typical fee is immense compared to that of traditional money managers, who often charge a management fee of 50 basis points or less with no incentive component. The relative size of the fees can vary across HFs, with established and consistently successful funds able to charge more. Examples of exorbitant incentive fees abound, with incentive fees as high as 50 percent for SAC Capital Partners or Renaissance Technologies’ 5 percent management and 44 percent incentive fees.

Additionally, HFs will sometimes receive an incentive fee only on returns in excess of a specified hurdle. The most common hurdle is a benchmark index, often 3-month U.S. Treasury Bills or LIBOR. A HF may also be subject to a high watermark, whereby it cannot charge an incentive fee if the value of the assets under management is below a previous high at the end of the period.

On top of what is paid to the HFs in which they invest, FOFs charge substantial fees. As with HFs, individual FOF fee structures vary, but the industry norm is 1 a percent management fee and a 10 percent incentive fee (“1 and 10”). Combined, the HF and FOF incentives fees can eat an extremely large portion of absolute returns.
1.4 Incentive Fees as Options

Although management fees are straightforward to value, the incentive fees are more problematic. As the incentive fees only matter when the fund makes an absolute return, the manager is participating in the potential upside of the fund without being exposed to the underperformance (at least from an incentive fee standpoint). Taking 20 percent as a starting point, below is a representation of the payoff diagram for the annual performance of a fund given the gross of incentive fee return:

As is apparent from the diagram, the incentive fee payoff for a manager is a call option on the assets under management with a strike price equal to the hurdle rate. Despite reputational costs of underperformance, this fee structure introduces incentive problems between the FOF manager and the investor. Since the FOF manager is selling an option to the investor the value of this option is higher with more volatility [7], the “heads, I win, tails you lose” effect.

In a FOF the investor directly pays incentive fees to both the HFIs based on individual performance and the FOF based on aggregate performance net of individual HF fees. As a result the investor is short both a portfolio of incentive fee options to the HF managers and an option on the portfolio net of HF fees to FOF. Because HF performance will vary, some funds will presumably outperform while others might see low or even negative returns. This different fee structure can lead investors in FOFs to pay large incentive fees for a portfolio that exhibits underperformance. Since FOFs take a 10 percent incentive fee for the portfolio on top of individual managers, the investor is ceding a substantial part of the upside.
even when there is relatively uniform positive performance.

A simplified example shown in Table 1 would be a FOF that invests $1 million equally in three HFs. At the end of one period, the first HF has a positive 20 percent return, the second a positive 10 percent, and the last a negative 30 percent return. For the sake of simplicity, let us ignore management fees. Gross of all fees, performance of the portfolio would be zero. However, given the structure of the incentive fees for individual funds, the first two funds would receive 4 and 2 percent respectively, while the third would receive nothing. Therefore, the total return of the portfolio net of HF incentive fees would be negative 2 percent. The payment of incentive fees by the investor due to the bundling of HFs with individual incentive fees is a drag on total performance. Throughout the paper we attempt to measure the effect with “deadweight cost,” defined as the difference between the expected value of the sum of fees paid to the individual HFs and the expected value of an incentive fee paid on the pooled assets.

<table>
<thead>
<tr>
<th></th>
<th>Fund 1</th>
<th>Fund 2</th>
<th>Fund 3</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Return</td>
<td>20%</td>
<td>10%</td>
<td>-30%</td>
<td>0%</td>
</tr>
<tr>
<td>Incentive Fee</td>
<td>4%</td>
<td>2%</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>Net Return</td>
<td>16%</td>
<td>8%</td>
<td>-30%</td>
<td>-2%</td>
</tr>
</tbody>
</table>

Table 1: An example illustrating the effect of incentive fees on returns. The FOF consists of 3 HFs with an overall 1-yr gross return of 0% and net return of -2%.

The payment of high total FOF fees despite modest returns is more than academic. In Dr. Rosenfeld’s experience at Paloma Partners, he saw a Paloma-managed FOF go from gross returns of 8 percent to net returns of 1 percent. We look to examine this phenomenon, by valuing the options that the investor cedes to the HF and FOFs, and exploring the effects of correlation, volatility, and number of HFs in a FOF on fees paid.

2 Model

Let $V_i$ be the value of an investment made in HF $i$, $i \in \{1, 2, \ldots, n\}$. Let $\mu_i$ be the expected continuously compounded return per year and $\sigma_i$ be the standard deviation per year. We assume that the $V_i$ evolve according to a multidimensional geometric Brownian motion, which can be specified, for $i \in \{1, 2, \ldots, n\}$,
by,
\[ dV_i = \mu_i V_i dt + \sigma_i V_i dz_i \]  
(1)
where, \( z_i \) is a standard single-dimensional Weiner process (Brownian motion) that is correlated with \( z_j \), and the correlation coefficient is \( \rho_{ij} \).

By Ito’s Lemma, we have that \( d \ln V_i \) follows a generalized Weiner process,
\[ d \ln V_i = (\mu_i - \frac{\sigma_i^2}{2}) dt + \sigma_i dz_i. \]  
(2)
So,
\[ \ln V_i(T) - \ln V_i(0) = (\mu_i - \frac{\sigma_i^2}{2}) T + \sigma_i \sqrt{T} \epsilon_i \]  
(3)
where \( V_i(T) \) and \( V_i(0) \) are the values of the investment at time \( T \) (in years) and at the present, respectively; \( \epsilon_i \) is standard Normal random variable that is correlated with \( \epsilon_j \), and the correlation coefficient is \( \rho_{ij} \). In fact, the \( \epsilon_i \)'s are assumed to be jointly Gaussian. In vector notation, let \( \epsilon = \begin{bmatrix} \epsilon_1 & \epsilon_2 & \ldots & \epsilon_n \end{bmatrix}' \), then \( \epsilon \sim N(0, \Sigma) \), where,
\[ \Sigma = \begin{bmatrix}
1 & \rho_{12} & \ldots & \rho_{1n} \\
\rho_{12} & 1 & \ldots & \rho_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{1n} & \rho_{2n} & \ldots & 1
\end{bmatrix}. \]  
(4)

Now, define \( r_i \) to be the continuously compounded annual return for HF \( i \),
\[ r_i = \frac{1}{T} \ln \frac{V_i(T)}{V_i(0)}. \]  
(5)
By equation (3), \( \ln(V_i(T)/V_i(0)) \) is Normally distributed, and hence so is \( r_i \). In vector notation, we have that
\[ \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} \sim N \left( \begin{bmatrix} \frac{\mu_1 - \sigma_1^2}{2} \\ \frac{\mu_2 - \sigma_2^2}{2} \\ \vdots \\ \frac{\mu_n - \sigma_n^2}{2} \end{bmatrix}, \frac{1}{T} \begin{bmatrix} \sigma_1 & \sigma_2 & \ldots & \sigma_n \\ \sigma_2 & \sigma_1 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \sigma_n \\ \sigma_n & \ldots & \sigma_2 & \sigma_1 \end{bmatrix} \Sigma \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{bmatrix} \right). \]  
(6)
While an assumption of Normal HF returns may seem to limit the real-world applicability of our model, our results and analysis is still directionally informative because HFs spend most of their time in the relatively stable regime of low correlation and low standard deviation. As a result most fees are paid under this regime where correlations and standard deviations are sufficient for describing the distribution.
2.1 Returns, Fees and Sharpe Ratio

As an individual HF fee structure, we have assumed a fairly standard “2 and 20” or a 2% management and 20% incentive fee. For the FoF we have assumed “1 and 10,” or a 1% management and 10% incentive fee. The investor directly pays the fees of the individual HFs and the FoF. A given HF receives incentive fees on that HF’s 1-yr returns in excess of its management fees. The FoF collects an incentive fee on its 1-yr returns in excess of individual HF fees and its own management fees. Regardless of returns both individual HFs and the FoF earn management fees. Investors pay all fees at the end of the year, and the management fees are based on total fund assets at that time (we might think of using initial or average assets, but the difference is negligible). Additionally, we have assumed that there is no hurdle rate, or performance benchmark, for incentive fees. Although it makes no difference in a single period simulation, there is also no high-water mark, or requirement that incentive fees are paid only when fund value exceeds previous highs.

We let $R_{i}^{GROSS}$ denote the annualized return for HF $i$, before all fees are taken out. That is,

$$1 + R_{i}^{GROSS} = e^{r_i T} = \frac{V_{i}(T)}{V_{i}(0)}.$$  

Then, at time $T$, after fees, $F_{i}(T)$, we have

$$V_{i}^{NET}(T) = V_{i}(T) - F_{i}(T) = V_{i}(T) - \beta_{i}V_{i}(T) - \gamma_{i}\max\{0, V_{i}(T) - \beta_{i}V_{i}(T) - V_{i}(0)\}.$$  

Here, $\beta_{i}$ is the management fee charged by HF $i$ and $\gamma_{i}$ is the incentive fee. Note that we have assumed here that there is no hurdle or benchmark. So, the net return on the investment in HF $i$, $R_{i}^{NET}$, is

$$1 + R_{i}^{NET} = \frac{V_{i}^{NET}(T)}{V_{i}(0)}.$$  

Using equation (9) together with the fact that $V_{i}(T) = (1 + R_{i}^{GROSS})V_{i}(0)$ and rearranging, we have that

$$R_{i}^{NET} = R_{i}^{GROSS} - \beta_{i}(1 + R_{i}^{GROSS}) - \gamma_{i}\max\{0, (1 - \beta_{i})R_{i}^{GROSS} - \beta_{i}\}.$$  

Next, suppose that a fund of fund has $V_i(0)$ invested in each of the $i$ HFs. That is, its initial investment is $V_{FOF}(0) = \sum_{i=1}^{n} V_{i}(0)$. Then, at time $T$, before all fees, the value of the investment is $V_{FOF}(T) = \sum_{i=1}^{n} V_{i}(T)$. So, the return on this portfolio gross of all fees is obviously $\sum_{i=1}^{n} \alpha_{i} R_{i}^{GROSS}$, where $\alpha_{i} = V_{i}(0)/V_{FOF}(0)$.
At time $T$, the due fees will be paid to each of the HFs, and the return of the FOF gross of its own fees, $R_{FOF}^{GROSS}$, will be,

$$1 + R_{FOF}^{GROSS} = \frac{V_{FOF}(T) - F_{HF}(T)}{V_{FOF}(0)}$$

$$= \sum_{i=1}^{n} \frac{V_i(T) - \sum_{i=1}^{n} F_i(T)}{\sum_{i=1}^{n} V_i(0)}. \quad (12)$$

Clearly, we have that $R_{FOF}^{GROSS} = \sum_{i=1}^{n} \alpha_i R_{i}^{NET}$. So, substituting from equation (11),

$$R_{FOF}^{GROSS} = \sum_{i=1}^{n} \alpha_i R_{i}^{GROSS} - \sum_{i=1}^{n} \alpha_i \beta_i(1 + R_{i}^{GROSS}) - \sum_{i=1}^{n} \alpha_i \gamma_i \max\{0, (1 - \beta_i) R_{i}^{GROSS} - \beta_i\}. \quad (13)$$

In this sum, the second term is due to HF management fees and the last term is due to the HF incentive fees.

Finally, the FOF investor also pays to the FOF at time $T$ a fee, $F_{FOF}(T)$, consisting of $\beta_{FOF}$ in management fees and $\gamma_{FOF}$ in incentive fees. So, the net of all fees return to the investor, $R_{FOF}^{NET}$, will be

$$1 + R_{FOF}^{NET} = \frac{V_{FOF}(T) - F_{HF}(T) - F_{FOF}(T)}{V_{FOF}(0)}, \quad (15)$$

which, after substituting equation 12 and

$$F_{FOF}(T) = \beta_{FOF}(V_{FOF}(T) - F_{HF}(T)) + \gamma_{FOF} \max\{0, V_{FOF}(T) - F_{HF}(T) - \beta_{FOF}(V_{FOF}(T) - F_{HF}(T)) - V_{FOF}(0)\},$$

and simplifying becomes,

$$R_{FOF}^{NET} = R_{FOF}^{GROSS} - \beta_{FOF}(1 + R_{FOF}^{GROSS}) - \gamma_{FOF} \max\{0, (1 - \beta_i) R_{FOF}^{GROSS} - \beta_i\}. \quad (16)$$

Here, we point out two random variables that will be used subsequently.* Specifically, combining equations (12) and (15), we have that

$$R_{FOF}^{GROSS} - R_{FOF}^{NET} = \frac{F_{FOF}(T)}{\sum_{i=1}^{n} V_i(0)}, \quad (18)$$

and similarly, using equation (15) with the fact that $\sum \alpha_i R_i = \sum V_i(T) / \sum V_i(0)$, we get that,

$$\sum \alpha_i R_i - R_{FOF}^{NET} = \frac{F_{HF}(T) + F_{FOF}(T)}{\sum_{i=1}^{n} V_i(0)}. \quad (19)$$

*As we will describe in the next section, we can relate the expected values of these random variables to the present value of the incentive fees that is computed by viewing the incentive fees as options.
Furthermore, because rational investors should care about risk-adjusted returns and not fees, we also make use of the Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{E(R_{\text{NET FOF}}) - R_f}{\sigma_{\text{NET FOF}}},$$

(20)

where $R_f$ is the annualized risk-free rate. Although the Sharpe ratio’s use of variance as a proxy for risk diminishes its value in the fat-tailed world of HFs, it remains a valid metric under the Normality assumptions of our model.

2.2 Valuation of Incentive Fees as Options

Consider the incentive fee that will be paid to HF $i$ at time $T$,

$$F_i^{INC}(T) = \gamma_i \max\{0, V_i(T) - \beta_i V_i(T) - V_i(0)\}$$

(21)

$$= \gamma_i (1 - \beta_i) \max\{0, V_i(T) - \frac{V_i(0)}{(1 - \beta_i)}\}.$$  

(22)

As noted earlier, the incentive fee payment looks like the payoff of a call option. Because we have assumed that the value of the HF is Log-normally distributed (equation (3)), we can use Black-Scholes formula to value this option, where the strike price is $V_i(0)/(1 - \beta_i)$.

Similarly, the incentive fee that is paid to the FOF at time $T$ is

$$F_{\text{FOF}}^{INC}(T) = \gamma_{\text{FOF}} \max\{0, (V_{\text{FOF}}(T) - F_{HF}(T)) - \beta_{\text{FOF}}(V_{\text{FOF}}(T) - F_{HF}(T)) - V_{\text{FOF}}(0)\}.$$  

(23)

Here, the option is on the FOF portfolio minus HF fees, that is, a portfolio that is long the HFs and short the call options that represent HF fees. Recall that

$$V_{\text{FOF}}(T) = \sum_{i=1}^{n} V_i(T),$$

(24)

and,

$$F_{HF}(T) = \sum_{i=1}^{n} \alpha_i (\beta_i V_i(T) + \gamma_i \max\{0, V_i(T) - \beta_i V_i(T) - V_i(0)\}).$$

(25)

First, even though we have assumed that $V_i(T)$ are Log-normal, it is not necessarily true that the sum, $V_{\text{FOF}}(T)$, is also Log-normal. Second, the nonlinearity in equation (25), the $\max\{0,\}$, in theory, further alters the distribution of $V_{\text{FOF}}(T) - F_{HF}(T)$.

In practice, a fast approach to value this option, which, except for the fees, looks like a basket option, would be to assume that the value of the underlying at time of maturity is Log-normally distributed,
with appropriate parameters ([8], page 541). Another approach is to assume that the individual assets’ values follow geometric Brownian motion, as we have done, and use Monte Carlo simulation. We chose the latter approach, as we describe in section 3.

As an aside, we note that we can relate the present value of the the incentive fee to the expected future incentive fee using the notion of certainty equivalents. We evaluate the value of the incentive fee using option pricing methods,

\[ C = \exp(-r_f T) \hat{E}(F^{INC}(T)), \]  

where \( \hat{E}(.) \) is the risk neutral probability measure. \( \hat{E}(F^{INC}(T)) \) is the certainty equivalent of \( E(F^{INC}(T)) \) in the risky measure. The ratio, \( E(F^{INC}(T))/\hat{E}(F^{INC}(T)) = 1 + R_{adj}, \) gives a “risk adjusted” opportunity cost, \( R_{adj}. \) And so the difference, \( E(F^{INC}(T)) - \hat{E}(F^{INC}(T)), \) is proportional to the “risk premium.”

2.3 Deadweight Cost

As mentioned in the Introduction, we are primarily concerned with the effects resulting from the individual HF fees paid by the investor. As a portfolio of short call options on individual HFs, these fees create a surprising amount of drag on returns, which in [2] is estimated at 100 basis points. In order to capture this effect, we use a “measure” called deadweight cost.

Deadweight cost is the difference between the present value of total incentive fees paid to individual HFs generating returns \( R_i, \) (equivalently, having value \( V_i(T) \) at time \( T \)), and the present value of the fee that would have been paid to a single fund with return \( \sum R_i, \) (equivalently, having value \( \sum V_i(T) \) at time \( T \)). This difference is a “measure” of how costly it is to have to pay incentive fees to each HF individually, in comparison to an alternative structure where the fee is paid on the return of the overall portfolio.

Suppose we invest 1$, so that \( \sum V_i(0) = 1. \) That is, we invest \( \alpha_i \) in each of the HFs. Further, we assume that \( \gamma_i = \gamma \) and \( \beta_i = \beta. \) Then, we have that the total incentive fees paid to the individual HFs is

\[
\sum_{i=1}^{n} F^{INC}_i(T) = \gamma \sum_{i=1}^{n} \max\{0, V_i(T) - \beta V_i(T) - \alpha_i\}. \tag{27}
\]

On the other hand, the incentive fee paid on the overall portfolio of HFs is

\[
F^{INC}_{ALT}(T) = \gamma \max\{0, \sum_{i=0}^{n} (V_i(T) - \beta V_i(T)) - 1\}. \tag{28}
\]

The deadweight cost is the difference in the present values of \( \sum_{i=1}^{n} F^{INC}_i(T) \) and \( F^{INC}_{ALT}(T). \)
Note that equation (27) represents the payoff of $n$ call options and equation (28) is the payoff on a portfolio of assets. So, the present values can be computed, as in option pricing, by discounting the expected values in the risk-neutral measure by the risk-free rate. We have that,

$$C_i = \exp(-r_f T) \hat{E}(F_i^{INC}(T)) = \exp(-r_f T) \gamma \hat{E}(\max\{0, (1 - \beta) V_i(T) - \alpha_i\})$$  \hspace{1cm} (29)$$

and

$$C_{ALT} = \exp(-r_f T) \hat{E}(F_{ALT}^{INC}(T)) = \exp(-r_f T) \gamma \hat{E}(\max\{0, (1 - \beta) \sum_i V_i(T) - 1\})$$ \hspace{1cm} (30)$$

where, $r_f$ is the risk-free rate, and $\hat{E}(\cdot)$ is the expected value with respect to a risk neutral probability measure.\footnote{Note that since we have assumed that the $V_i$ are a correlated Brownian motion process, we can apply the Black-Scholes formula to value these two options. Monte Carlo simulation gives the identical result.}

Finally, the deadweight cost is

$$\text{Dead Weight Cost} = \sum_{i=1}^{n} C_i - C_{ALT}.$$  \hspace{1cm} (31)$$

## 3 Simulation

We start by generating a sample of $N$ vectors drawn from the distribution of equation (6). For each sample, we convert $r_i$ to $R_{GROSS}^F$ using (7) and compute $R_{GROSS}^F$ and $R_{NET}^F$ using equations (14) and (17). We also compute $\sum \alpha_i R_i - R_{NET}^F$ and $R_{GROSS}^F - R_{NET}^F$.

To estimate expected value, $E(R_{GROSS}^F)$, we average the values of $R_{GROSS}^F$ computed for the $N$ samples. Similarly, we estimate $E(R_{GROSS}^F)$, $E(F_{FOF}(T))$ and $E(F_{FOF}(T) + F_{HF}(T))$. We also estimate the variances of the random variables by using the unbiased estimator. Specifically, let $X$ be the random variable whose expected value and variance we wish to estimate, and let $x_i, i \in \{1, \ldots, N\}$, be the generated sample values. Then,

$$\hat{\mu}_X = \frac{1}{N} \sum_{i=1}^{N} x_i,$$  \hspace{1cm} (32)$$

and

$$\hat{\sigma}_X = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \hat{\mu}_X)^2.$$ \hspace{1cm} (33)$$

Finally, having estimates of $E(R_{NET}^F)$ and $\sigma_{NET}^F$, we estimate the Sharpe ratio using equation (20).
Next, we value the incentive fees using Monte Carlo simulation. Consider a call option on an asset with value $V_i$ and strike price $K$. Recall that the present value of the option, $C$, with payoff at time $T$ equal to $\max\{0, V_i - K\}$, is,

$$C = \exp(-r_f T) \hat{E}(\max\{0, V_i - K\}),$$

(34)

where, $r_f$ is the risk-free rate, and $\hat{E}(\cdot)$ is the expected value with respect to a risk neutral probability measure. Rather than compute this probability measure and hence the expected value, an equivalent approach, when $V_i$ is geometric Brownian motion, is to use the risk-free rate as the drift parameter, $\mu_i = r_f$ in equation (1) [5, 8]. That is,

$$dV_i = r_f V_i dt + \sigma_i V_i dz_i.$$

(35)

Thus, when simulating, we can generate a sample of risk-neutral $V_i(T)$ according to

$$V_i(T) = V_i(0) \exp \hat{r}_i,$$

(36)

where,

$$\hat{r}_i = (r_f - \frac{\sigma_i^2}{2})T + \sigma_i \sqrt{T} \epsilon_i.$$

(37)

We then compute the payoffs from the option at time $T$, take the mean to get the expected payoff in the risk-neutral world, then discount to the present using the risk-free rate.

To value the incentive fees on each of the HFs, the FOF, and a portfolio of HFs, we start with the $N$ samples of $r_i$ that we have already generated. From those, we generate samples of $\hat{r}_i$, using the fact that

$$\hat{r}_i = r - (\mu_i - r_f).$$

(38)

Then, we substitute in (36) to generate samples of risk-free $V_i(T)$ and then compute $F_i(T)$. Using equations (21), (23) and (28), we compute the payoffs of our options for each of our $N$ samples. We then calculate the mean and discount it at the risk-free rate to obtain the desired value of the incentive fee “options.”

3.1 Model Parameters

We ran all our simulations on MATLAB. When we used historical returns, correlations, and variances from [3], we used $N=1,000,000$ samples drawn from the multi-variate Normal distribution. For the rest of our simulations, we used $N = 100,000$. 

11
We ran a single-period simulation lasting for one year, \( T = 1 \). While a multi-year simulation might tell us something about the distribution of long-term returns, it would provide little additional information concerning the issues we wish to understand about the fee structure.

We have taken the risk free rate, \( r_f \), to be 5%. While the proper value for the risk free rate is debatable, this does not impact our conclusions.

For our simulations involving historical data, in order to estimate the expected returns for the different fund styles, we adjusted the net returns given in [3]. Our rough calculation of gross fees simply adds back the incentive fee as a percentage of the net returns and then adds back the management fee, \( R_{i}^{\text{GROSS}} = 1.25 R_{i}^{\text{NET}} + 0.02 \). This leads to a slight but unimportant underestimate of gross returns. Although we do not explicitly account for survivorship bias, we benefit from the Lo paper’s adjustments to minimize its effects on observed return and correlation.

For our the rest of our simulations, we study the effects of the correlations between HFs and their volatilities on the Sharpe ratio and deadweight cost. We also study the effect of number of funds, \( n \), on these two quantities. For these simulations, we use \( \mu_i = \mu = 15\% \). Unless it is varied, \( \sigma_i = \sigma = 10\% \). Unless it is varied, \( n = 14 \). The correlation matrix is chosen to be

\[
\Sigma = \begin{bmatrix}
1 & \rho & \ldots & \rho \\
\rho & 1 & \ldots & \rho \\
\vdots & \vdots & \ddots & \vdots \\
\rho & \rho & \ldots & 1
\end{bmatrix}
\]

where \( \rho \in [0,1] \). We choose this structure because it is a simple structure that we can parametrize by one variable, \( \rho \). Note that a portfolio of assets with this \( \Sigma \) and \( \sigma_i = \sigma \) will have a variance of \( \sigma_p^2 = \sigma^2 \left( \frac{1}{n} + \frac{n-1}{n} \rho \right) \).
4 Results

Using real data for the returns, volatilities, and correlations for individual HF's of different styles, Monte Carlo simulation reveals that the expected value of total HF and FOF fees on a typical FOF equal roughly 7% of assets and that the incentive fees on individual HF's alone sum to nearly 2.5% of assets. We calculate the expected value of these fees by constructing a portfolio comprising 14 HF's with one corresponding to each style in the CSFB/Tremont hedge fund indexes. Although the number of HF's in a real FOF is often larger than 14, we will show that this has a rather modest effect on total fees and no effect on individual HF fees. Running the standard set of 1,000,000 trials gives us the results displayed in Table 2 and Figure 1. Despite the surprising magnitude of the total fees, they remain consistent with Dr. Rosenfeld's previously mentioned experience at Paloma where fees reduced gross returns of 8% to net returns of 1%.

<table>
<thead>
<tr>
<th></th>
<th>Historical $\mu$, $\sigma$, $\Sigma$</th>
<th>$\mu = 0.15$, $\sigma = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\rho = 0.25$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\rho = 0$</td>
</tr>
<tr>
<td>$\sum \alpha_i R_{i, gross}$</td>
<td>0.1432 0.0468</td>
<td>0.1618 0.0638 0.1620 0.0311</td>
</tr>
<tr>
<td>$R_{gross}^{FOF}$</td>
<td>0.0943 0.0370</td>
<td>0.1099 0.0511 0.1100 0.0249</td>
</tr>
<tr>
<td>$R_{net}^{FOF}$</td>
<td>0.0750 0.0330</td>
<td>0.0889 0.0457 0.089 0.0222</td>
</tr>
<tr>
<td>$C_{FOF}$</td>
<td>0.0018 0.0023</td>
<td>0.0014</td>
</tr>
<tr>
<td>$\sum C_i$</td>
<td>0.0098 0.0109</td>
<td>0.0109</td>
</tr>
<tr>
<td>Deadweight</td>
<td>0.0031 0.0032</td>
<td>0.0048</td>
</tr>
</tbody>
</table>

Table 2: Returns, fee value, risk-neutral fee option value, and deadweight cost for FOF's with historical and model parameters.

In the following pages, we go on to examine the effects of the correlation between funds, the volatility of individuals funds, and the number of funds in a FOF on its total fees, deadweight cost, and Sharpe ratio. Unless otherwise stated, we use a portfolio of 14 funds each having a return mean of 15% and standard deviation of 10%.
Figure 1: The numbers in Chart 1(a) are based on historical average returns, correlations and variances for the 14 hedge fund strategies in [3]. The calculations of Chart 1(b) use the risk-neutral valuation framework and assumes no ex-ante belief about management skills.

4.1 Correlation

You might think the expected value of total fees paid on individual HFIs would decrease with rising correlation. Taking the extreme case of perfect correlation, all funds would have either positive or negative aggregate returns from a given period. The sum of the incentive fees on the individual funds would converge to a comparable incentive fee on the underlying assets, and you would never pay an incentive fee on high-performing funds even while others faltered. Figure 2, which shows the incentive fees paid in individual trials to individual HFIs for $\rho = 0, 0.25, \text{and } 1$, would seem to reinforce this belief—surely it must be costly to be above the $\rho = 1$ line. This, however, is not the case, and correlation actually affects only the volatility and not the expected value of total HF fees. The values of expected returns and standard deviations, as well as the values of the incentive fees, for this simulation, are shown in Table 2.

To understand what actually happens, we again think of the fees as a portfolio of short call options on the underlying funds. Our equation for the sum of these fees is

$$\sum_i \gamma_i \max\{0, (1 - \beta_i)R_i^{GROSS} - \beta_i\}.$$ 

This is clearly independent of the correlation between funds, which affects only whether fees on all
Figure 2: Plot of total HF fees vs. gross returns for 100,000 individual trial runs.

individual HFs are likely to be paid on the same trial or spread over many trials. In our figure the few
\(\rho=1\), high-return, high-fee trials occur just frequently enough to compensate for the low-correlation trials
above the line.

While higher correlation does not affect the expected value of total HF fees, it clearly increases the
standard deviation of both returns and the sum of these fees. The latter effect occurs because the
investors become more likely to pay fees on either all HFs or none.

4.2 Correlation and Volatility

Consistent with the paper by Brown, Goetzmann, and Liang [2], our findings in Figure 3 show deadweight
cost that is on the order of 60 basis points (\(\sigma=0.1, \rho=0\)) and increases with lower correlation and higher
volatility. It is important to remember that deadweight cost is not the same as expected value of fees.
Instead it is a relative metric that takes the sum of the fees paid on individual hedge funds and subtracts
the fees that would have been paid on the sum of returns (see equation 31).

Increasing correlation decreases deadweight cost because it increases the value of the second term
without affecting the first. Specifically increased correlation increases the volatility of the combined
portfolio’s return, \(\sum \alpha_i R_{iGROSS}^{GROSS}\), thus increasing the value of the hypothetical call on that return. As
mentioned above the value of the portfolio of short call options equals the short call on the portfolio in
the case of perfect correlation, and the deadweight cost goes to zero.
For volatility, increased standard deviation of returns leads to higher value for both the sum of the calls on individual HF returns and the call on the single return. Because the call on the single return pools the volatility of n assets, it increases less than the sum of the calls on individual HF fees. This gives us the observed increase in deadweight cost with rising volatility. As is evident from the equal vertical spacing between points on our diagram, option vega of the deadweight cost is constant for a given $\rho$. This is an expected result of our model assumptions. Ultimately, the benefit the investor derives from lower volatility leading to lower fees is universal across incentive fee structures based on return.

### 4.3 Correlation and Number of Funds

Examining deadweights cost over a variety of correlations and numbers of funds in Figure 4, we find that it is again lower for high correlation (zero for perfect correlation) and that it increases with the number of HFs in a FOF. Correlation affects deadweight cost here through the same mechanism as in Figure 3. Similarly, while having more hedge funds does not affect the expected value of individual hedge fund incentive fees, it decreases the value of the call on the combined portfolio’s returns by reducing their
An Analysis of Fees in Funds of Funds

Figure 4: Deadweight cost for FOFs with differing correlation and number of HF s.

volatility. This results in greater deadweight cost when a FOF has more funds. Of course it is again imperative to remember that the greater deadweight cost is relative to a fee on the combined portfolio’s returns—the number of funds actually has no effect at all on the expected value of the total HF fees. Especially if correlation between funds is low, more hedge funds do, however, reduce the volatility of the portfolio, thus lowering the value of the FOF’s incentive fee.

Ignoring administrative costs, Figure 5 implies significant but rapidly diminishing benefits from diversification, which coincides with what is observed empirically[12]. Thus within the classical mean-variance framework, diversification significantly reduces volatility, without affecting the expected return, and ultimately leads to a higher Sharpe ratio. That said, HF s often have strongly skewed and periodically correlated returns, and the standard deviation and Sharpe ratio do not account for non-normality and time-varying correlation. However, within our framework, the Sharpe ratio (Figure 5) behaves very much as it would if there were no fees at all, and the most we can infer by looking at the Sharpe ratio is that management and incentive fees alone do not play a central role in determining the optimal number of funds in a FOF.
5 Discussion and Conclusions

To recap, we use two equivalent approaches for evaluating the impact of the traditional fees structure on the performance of FOFs. The first approach relies on the classical options framework, where the incentive fees of the individual HFs in the portfolio are treated as call options on the underlying securities minus asset-based expenses. The incentive fee on the entire FOF portfolio is thought of as a call option on the basket of assets (i.e., underlying HFs) minus asset-based expenses. To value the HF and FOF options, we use Monte-Carlo simulation of return outcomes while the second approach relies on the Monte-Carlo simulation of return outcomes and the respective incentive fees at each level of the portfolio structure (i.e., individual HF fees, FOF fees).

While FOF structure may provide the sought-after diversification effect, we observe that this comes at a cost of multi-layered incentive fees and asset-based expenses. Confirming our intuition, we find that the underlying HF incentive fees has a significantly larger negative impact on the overall portfolio than either the FOF incentive fee or any of the asset-based expenses within the portfolio structure. This is in line with previous literature [2].
We also determine that, while changing the number of underlying funds and the inter-fund correlation substantially alters the variability of the total incentive fees paid to the underlying funds, it does not affect the expected value of the total incentive fees paid to underlying HFs. This means there is no absolute cost in terms of excess fees to increasing the diversification of your FOF portfolio. However, there is certainly an additional cost in terms of manager search and monitoring expenses. Also, as Lhabitant and Learned [10] report, when you depart from the classical mean-variance framework and consider effects such as skewness and kurtosis, diversification becomes less of a proverbial “free lunch” due to a tradeoff between profit potential and reduced probability of loss. However, a multi-strategy fund structure possesses its own set of problems resulting from a unique set of agency issues. Unlike a typical FOF, a multi-strategy fund does not take an arms-length approach to its underlying managers. Each manager belongs to the same organization, which tends to create problems of political nature when it comes to manager termination, compensation and capital allocation. A quintessential example of such problems is the case of Amaranth Advisors mentioned in the introduction. Amaranth claimed to take a multi-strategy approach, but in fact lost more than half of its capital due to a highly leveraged bet in the natural gas futures market placed by a single, “super-star” manager.

Brown, Goetzmann and Liang [2] propose an alternative to the traditional FOF fee arrangement wherein a FOF fully absorbs the individual HF incentive fees. The FOF manager could then hedge the underlying incentive fee exposure. The arrangement could be made revenue-neutral for the FOF manager if this additional cost is reimbursed either in a form of a higher asset-based expense or through a larger incentive fee. We also argue that for investors, compensating the FOF manager with a higher asset-based fee is superior to the ramped-up incentive fee because it avoids increasing the FOF’s desire for volatility. Also, such a compensation arrangement would provide substantial benefit to the investor by making the cost structure more transparent. Additionally, the desire to minimize the cost of hedging would encourage a FOF to better understand the underlying HF strategies resulting in an increased effort to perform due-diligence on the underlying funds.

In a final rule effective July 31st, 2006, the SEC has already increased transparency by requiring FOFs to disclose expected expenses incurred from investments in individual HFs through “Example” tables. These tables, however, give only expenses as calculated from historical returns. As a result the newly required information is not fully indicative of the fees investors will pay. [11]

More effective ways to manage investor costs should involve controlling them at the individual HF level. To be successful at this, a FOF should attempt to leverage its power over individual HFs. Large
and established FOFs may have a certain amount of bargaining power over newer, smaller HFs, but they will have a very tough time negotiating with perennial powerhouses such as Citadel or SAC Capital. One way to control individual manager costs is the use of HF incentive fees based on factors beyond returns. An example of such an arrangement is a binary incentive fee structure that pays zero incentive fees if the realized volatility of intra-period returns exceeds a certain threshold, but pays the full incentive fee otherwise. This arrangement should incentivize individual managers to reduce their volatility, which in turn will reduce the overall cost of incentive fees. A potential drawback of this arrangement is its reliance on inter-period valuation of portfolio holdings for the measurement of return volatility. This creates an incentive for the HF manager to smooth out period to period valuation for holdings that are not frequently traded. In such a case, the valuation services of an independent third party may be required. A related arrangement might focus on volatility-adjusted reward to avoid punishing managers generating both higher volatility and higher returns.

We may also consider improving the quality of returns delivered to the investor for the given cost of the incentive fees. A way to do this is to imposing a longer time-frame for the incentive fee assessment. As was discussed by Kritzman [9], having a longer time-frame improves the statistical significance of the excess returns generated by the HF manager. While increasing the time-frame makes the option value of the incentive fee higher, we can scale this down by lowering the incentive fee percentage.

Alternatively, the use of high watermarks may better align the interests of the investors and HFs. As the fund asset level breaches the high watermark, the incentive fee falls to zero and does not begin to accrue until the high watermark level is once again exceeded. This feature reduces the cost of the incentive fees to the investor. In theory, this forces the manager to control volatility as it nears the high watermark. On the other hand, if the fund value falls below the high watermark, the manager has an incentive to raise the fund’s volatility. Goetzmann, Ingersoll, and Ross [6] have studied these and other effects of high watermarks on the HF compensation costs in great detail. An additional problem with a practical application of the high watermark provision is that a powerful HF can always renegotiate (scale-down) the high watermark if breached and a HF with weaker bargaining power can liquidate the fund altogether and simply start a brand new one.

Finally, requiring the manager to have a substantial portion of his wealth invested in the HF alters the incentives of the manager. While the manager benefits from successful investment decisions, he also suffers from bets gone wrong. Such a payoff structure forces the HF manager to control risks and minimize downside volatility.
To sum everything up, our examination of various alternatives demonstrates that it is not possible to improve the costs of investing in a portfolio of HF funds associated with managers’ incentive fees by varying parameters such as the number of funds or inter-fund correlations. Some of the alternative cost-control techniques involve imposing additional constraints on the individual HF managers. However, an important issue to consider is the adverse selection that may result from imposing additional constraints on the HF managers. As the bargaining position of an average HF deteriorates, the only ones willing to collaborate with FOFs will be those with either inferior or unproven investment skill. Thus, in the market where competition for investment skill is fierce and the supply thereof is scarce, a skilled hedge fund manager will tend to be in a good position to extract a lion’s share of profits from the providers of capital.
Acknowledgements

We wish to thank Dr. Rosenfeld for posing the interesting problem that we have studied here and for the important discussions, interesting feedback, and patient guidance. We are indebted to Professor Stephen I. Brown (NYU), who generously shared his time and thoughts on this subject. We also wish to thank Mr. Kritzman for providing us with direction and references. Finally, we thank Ms. Anna Obizhaeva for her time and help.

References


